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Optimal Portfolio Construction Using N – Assets Mean – Variance Portfolio Model: Study of Four Etfs of BSE

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Abstract: *A portfolio can be defined as a basket or collection of similar or different class of assets with the objective of maximizing returns and minimizing risk. Harry Markowitz's Modern Portfolio theory lays strong emphasis on portfolio diversification which enables the investor to spread the risks throughout the assets in the portfolio. This does not happen intuitively; rather it requires extensive and exhaustive research. The decision on investments is arrived through the process of portfolio optimization. This research aims to create an optimal portfolio comprising of four ETFs (Exchange Traded Funds) listed on BSE. The data collected for the research is secondary data of monthly prices of ETFs listed on BSE and is for the period Jan 2012 to June 2017. N – Asset Mean Variance Portfolio model has been employed to compute Variance – Co – Variance Matrices and Co – relation Matrices. The quantum of allocation in each asset depends on the risk and return characteristics of the asset. GRG – Non Linear Optimization Method has been employed to arrive at the quantum of allocation in each of the ETFs to create an optimal portfolio.*
Keywords: *Risk and Return; Portfolio Optimization; Mean – Variance; Asset Allocation*

I. Introduction

Every Investor who thinks rationally is deeply interested in maximizing his returns and minimizing his risks of the investments he/she has undertaken. That's precisely the reason he/she creates a portfolio. A portfolio can be defined as a basket or collection of similar or different class of assets with the objective of maximizing returns and minimizing risk. Securities consist of varying degrees of risk. Maximum number of investors hold on to multiple assets with the expectation that if one yields negative returns, the other assets will compensate and insulate the investment from extreme losses. The key aim of portfolio construction is to spread or in other words diversify risk by putting the eggs in different baskets. Thus the primary goal of having a portfolio is to generate maximum return at a particular level of risk or ensure minimum risk at a given level of return. Such portfolio(s) are known as optimal portfolios.

Financial Portfolio

A portfolio can be defined as a basket or collection of investments made in financial assets with the aim of minimizing risk through diversification. It can be termed as combination of securities, which are owned by the investor. It is the complete holdings in terms of volume and value owned by an individual. Portfolios can be built through inclusion of different asset classes such as debts, derivatives, equities in order to reduce risks. Employing a combination or mix of securities, the risk is far less than it would have been had the person invested in individual securities. The primary goal of portfolio construction is to ensure maximum returns at a particular level of risk or minimum level of risk.

Why Portfolio Management?

Portfolio management is a progressive course of action that includes formulation of investment goals, evaluation of investment atmosphere and situation, choice of appropriate securities and asset classes for investment, allocation of assets for portfolio construction, monitor and supervise the portfolio performance during the investment horizon, review the portfolio mix as and when required periodically. Prior to 1950s, Portfolio management process was not systematic and organized due to lack of scientific evaluation and analysis. The process gained significant traction after Harry Markowitz formulated the Modern Portfolio theory with Mean – Variance as a principle yardstick and as a scientific foundation for portfolio management. Exhaustive research has been carried out in the domain, which has lead to formulation of novel concepts aimed in the direction of refinement of portfolio management processes. The process has become highly professional with companies such as J.P Morgan, ICICI, Motilal Oswal, Franklin Templeton etc providing specific Portfolio Management Services to wealthy individuals through incessant monitoring of portfolios. However such services are beyond the realm and scope for common investors. Therefore the average investor has laid immense emphasis on building portfolios in capital markets

Modern Portfolio Theory

Modern Portfolio theory widely known as “portfolio theory” or “theory of investment “was promulgated by Harry Markowitz in 1952. He introduced the idea of diversification was also aided its quantitative development. Markowitz employed statistical techniques and analysis in order to come up with optimal allocation within given portfolios. The guidelines for allocation of available funds optimally in diverse number of securities were also formulated by him.

The theory propounded by Markowitz enables the choice of optimal or most efficient portfolio amongst numerous portfolios consisting of securities and asset classes. The model exhibits means to diminish risks from the investors’ perspective. The portfolio model has its foundations on mean returns or expected returns and variance (standard deviation) of diverse number of portfolios. This theory pertains to risk and return of portfolios instead of individual scrips. According to modern portfolio theory, degrees of risk and return vary across different portfolios. Markowitz proved that the primary objective behind investing in multiple securities is mitigation of risk. Markowitz theory can be regarded as the first methodical approach towards in alleviation of dilemma experienced by investors. He was awarded a Nobel Prize in Economics in 1990 for the revolutionary approach.

Investors endeavour to fulfill contradictory goals of low risk and high returns. In to order address this primary concern, Markowitz conceptualized a model that employs parametric optimization technique. This model was adequately generic in nature, so that it can be applied to a broad range of practical circumstances and at the same time, it was easy enough for theoretical evaluation and numerical computation.

II. Review Of Literature

OPTIMAL PORTFOLIO CONSTRUCTION WITH MARKOWITZ MODEL AMONG LARGE CAP’S IN INDIA

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The objective of this research paper was to build an effective portfolio comprising of large cap corporations. This research endeavour helps us to gain knowledge about the performance of certain companies that are a part of NIFTY – FIFTY Index. The research was undertaken on 15 large cap companies from NSE NIFTY – FIFTY. Financial data for the last four years was gathered for companies across different sectors namely: Information Technology, Energy, FMCG, Banking, Infrastructure, Pharmaceuticals etc. The research employed Sharpe’s Single Index model and it suggested the best stocks to invest in amongst the selected large cap companies.

A NOTE ON APPLYING THE MARKOWITZ PORTFOLIO SELECTION MODEL AS A PASSIVE INVESTMENT STRATEGY ON THE JSE

AJ du Plessis and M Ward* Investment Analysts Journal – No. 69 2009

The researchers in this paper employed Mean – Variance Portfolio theory to companies listed on Johannesburg Securities Exchange (JSE) in order to identify companies that can be a part of optimal portfolio. Weekly financial data of 11 years of the top 40 JSE listed companies was collated to build an optimized portfolio. The optimal portfolio was monitored and rebalanced regularly to reflect the current situation. The returns of the portfolio was compared against other portfolio indices. The study showed that this strategy outperformed the market substantially during the period for which the data was collected.

PORTFOLIO SIZE AND DIVERSIFICATION EFFECT IN LITHUANIAN STOCK EXCHANGE MARKET

Vilija, Egle, Ras, a Engineering Economics, 23(4), 2012.

This paper laid emphasis on effect of diversification instead of portfolio effectiveness and efficiency. This research was based on everyday stock prices during the period 2009 – 2010 and it was undertaken on companies listed on Lithuanian Exchange. This paper constructed portfolios of various sizes in order to understand the effect of non – systemic elimination. The researchers evaluate the effect of diversification of simple portfolios vis – a – vis stock portfolios of diverse weightage. The effect of diversification is investigated by measuring the quantum of diversifiable risk elimination in terms of percentage. It also depends on the number of securities present in the portfolio. The research concluded that portfolios comprising lesser number of securities showed prominent and significant diversification effect between simple and differently weighted securities.

Research Gaps
Though Markowitz's Mean – Variance portfolio methodology has been employed in a variety of markets and on numerous asset classes, in my limited knowledge, it has not been applied to construct portfolios of Exchange Traded Funds (ETF) of BSE.

This leads us to the following research questions: -
a) What is the risk – return profile of the ETFs chosen?
b) What should be the weightage of each ETF for portfolio construction?

Research Objective
The aforementioned research questions lead us to the following objectives of this research: -

- To construct an optimal portfolio of Four Exchange Traded Funds (ETFs) of Bombay Stock Exchange (BSE) to maximize return and minimize risk.
- To determine the investment proportion of each of the Four selected Exchange Traded Funds from the perspective of profit maximization and risk minimization.

III. Research Methodology

Type of Research: - Analytical Research.

Data Collection: - The data collected for the research is secondary in nature. In this research paper four Exchange Traded Funds (ETFs) listed on BSE have been chosen. The monthly adjusted closing market price data for the last five years i.e 01st January 2013 to 31st December 2017 was downloaded from the website: www.bseindia.com. Thus there are about 60 data points for each ETF.

Data Formulas
The adjusted closing prices of the ETFs were converted to logarithmic values to compute the returns.

i.e ln(R_t/R_{t-1})
The returns were computed through determination of difference of first order between lognormal values of the price of the current month with the lognormal values of the price of the previous month. The distribution of data was evaluated through Descriptive Statistics.
In order to calculate variance of stocks daily return and index return, I used the following historical volatility formula.

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Ri - Raverage)^2$$

Where "σ^2" is a variance of daily share return. Ri is a daily return of share i. Raverage is the average daily return. N is a sample size (252) days.

To measure, how stocks vary together, standard formula for covariance can be used.

$$\text{Cov (X,Y)} = \frac{1}{n-1}\sum_{i=1}^{n}[(X_i - \overline{X}).(Y_i - \overline{Y})]$$

where the sum of the difference of each value X and Y from the average is then further divided by the total number of values minus one. The covariance calculation enables us to calculate the correlation coefficient, shown as:

$$\text{Correlation Coefficient} = \frac{Cov(X,Y)}{\sigma_X.\sigma_Y}$$

Where σ is the standard deviation of each asset/stock. However, if there are more than two financial assets in the portfolio, then correlation and covariance matrices are needed to solve equations. To calculate standard deviation of portfolio (position) the following formula is used:

$$\sigma_p = \sqrt{\sum_{i=1}^{n}(w_i^2.\sigma_i^2) + 2(\sum_{i=1}^{n}\sum_{j=1}^{n}(w_i.\sigma_i.w_j.\sigma_J.\rho ij)}$$

Where "σ_p" is a standard deviation of portfolio, "σ_i" is a standard deviation of stocks. Wi is a weight of stocks in a portfolio and Pij is a correlation coefficient between stocks i and j.

Here, since we have selected four ETFs, we shall use covariance and correlation matrices to calculate portfolio return and portfolio standard deviation (risk).

Further, keeping the objectives of Profit (Return) Maximization and Risk Minimization in mind,
Microsoft Excel Solver function has been used to determine proportion of investment to be made in each ETF in order to obtain optimal portfolio.

Data Analysis and Interpretation
Step 1: - Main Parameters (Table 1)
Step 2: - ETFs Chosen (Table 2)
Step 3: - Statistical Details (Table 3)
Step 4: - Determine Covariance and Correlation between the Assets (Table 4 and Table 5)
Step 5: - Proportion to be allocated to each ETF based on specific objective using Solver
Function (Table 6)

Table – 1 – Main Parameters

Parameter	**Details**
Exchange	Bombay Stock Exchange (BSE)
Type of Instrument	ETFs
Size of Historical Data	63 Months
Testing Period	November 2012 to January 2018

Table – 2 – ETFs Chosen

SL.No	**Name of the ETF**
1	BankBees
2	GoldBees
3	Kotak Gold ETF
4	N100 Nasdaq

Table – 3 – Statistical Details

ETFs	**BankBees**	**GoldBees**	**Kotak Gold ETF**	**N100 Nasdaq**
Minimum	-12.67%	-11.05%	-227.24%	-16.01%
Average	1.33%	-0.16%	-3.91%	1.94%
Maximum	16.56%	11.48%	12.01%	19.88%
Variance	0.41%	0.18%	8.36%	0.44%
S.D	6.39%	4.21%	28.91%	6.62%

Table – 4 – Variance – Covariance Matrix

	BankBees	**GoldBees**	**Kotak Gold ETF**	**N100 Nasdaq**
BankBees	0.004078974	-0.00094009	-0.000409997	-0.000169889
GoldBees	-0.00094009	0.001773755	0.000361937	0.00010148
Kotak Gold ETF	-0.000409997	0.000361937	0.083599999	-0.000996494
N100	-0.000169889	0.00010148	-0.000996494	0.004376808

Variance – Covariance employs matrix. As we can see from the formula for portfolio measurement, standard deviation of portfolio calculation warrants computation of correlations of each asset and also covariance between them. Also application of variance covariance matrices is a pragmatic method of computing standard deviation of portfolio.
In this research, the objective is to show, how the parametric methodology utilizes variance and correlation matrices to calculate the variance, and hence standard deviation, of a portfolio. Covariance enables us to compute volatility of portfolios. Covariance values between stocks are multiplied by each shares weights and then added to find volatility of portfolio.

Table – 5 – Correlation Matrix

	BankBees	**GoldBees**	**Kotak Gold ETF**	**N100 NASDAQ**
BankBees	1	-0.34950017	-0.022202494	-0.040207954
GoldBees	-0.34950017	1	0.029722331	0.03642139
Kotak Gold ETF	-0.022202494	0.029722331	1	-0.052094613
N100	-0.040207954	0.03642139	-0.052094613	1

The strength of association between two variables is computed through correlation. This tells us the quantum in terms of percentage and direction in which the two variables move together. Volatility of a portfolio or the risk of a portfolio is lower than its individual assets volatility or risks. Therefore it is imperative to know the relation between variance of a portfolio and variances of assets. Correlation always has a value between -1 and +1 (Bozkaya, 2013). The coefficient of correlation may be computed by employing the covariance matrix that measures how mean returns of two companies vary or move together. Covariance matrix assists decision makers in deciding on the assets that move in the same direction or in the opposite direction (Bozkaya, 2013).

Application of MS Excel Solver to optimize portfolio allocation
MS Excel Solver was employed to optimize the portfolio and determine optimal proportion of allocation of funds amongst the four ETFs. Five different scenarios were tested and analysed for the purpose.

The five different scenarios have been listed below:
1) Maximize Return
2) Maximize return with the additional condition of keeping standard deviation below 4.212% (This is because amongst the four ETF standard deviations, this is the lowest (Goldbees ETF).
3) Minimize Risk (Standard Deviation).
4) Minimize Standard Deviation with the additional condition that the portfolio return should be minimum 1.937% (This is because amongst the four ETF returns, this is the highest (N100 Nasdaq).
5) Maximize Sharpe Ratio (Ratio of Mean Return to Risk (Standard Deviation)).

The optimal proportion for each of the aforementioned scenarios is presented in the table below:

Table – 6 (Scenario wise optimal proportion for allocation amongst the four ETFs)

	Scenario 1	**Scenario 2**	**Scenario 3**	**Scenario 4**	**Scenario 5**
	Max Ret	**Max Ret**	**Min St Dev**	**Min St Dev**	**Max SR**
Constraining Variable	**None**	**S.D<=**	**None**	**Expected Return>=**	**None**
Value		4.212%		1.937%	
BankBees Equity	0.00%	41.08%	29.87%	0.02%	40.71%
GoldBees Rshares	0.00%	6.67%	53.40%	0.00%	8.26%
Kotak Gold ETF	0.00%	0.00%	0.92%	0.00%	0.00%
N100 NASDAQ	100.00%	52.25%	15.80%	99.98%	51.02%
Sum of Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**
Mean Expected Return	1.9369%	1.5491%	0.5806%	1.9368%	1.5178%
S.D Expected Portfolio	6.6157%	4.2117%	2.6188%	6.6147%	4.1260%
Mean/S.D (Sharpe Ratio	0.29	0.37	0.22	0.29	0.37

IV. Discussion

As we can see, the optimal proportion of allocation is different for different scenarios.

In the first scenario, return is maximized, however at the cost of reasonably high standard deviation (risk) and low sharpe ratio (Not Desirable). The entire funds are to be invested in N100 NASDAQ Fund.

In the second scenario, due to the presence of additional constraint of keeping standard deviation below 4.212%, portfolio return has reduced to 1.5491%The third scenario minimizes standard deviation or risk to 2.6188%, however expected portfolio return too is at the lowest level.The optimal allocation in fourth scenario is identical to the first scenario, as the objective was to minimize risk under an additional condition of minimum expected return of 1.9368%. Here as well, it is suggested to invest almost all the money in N100 NASDAQ ETF.The fifth scenario with the objective of maximizing the return to risk ratio or Sharpe ratio has suggested huge chunk of allocation in BankBees and N100 Nasdaq with marginal allocation in GoldBees. This has a Sharpe Ratio of 0.37, which can be interpreted as “For every one hundred rupees invested, the expected portfolio return or 37 rupees.

V. Conclusion

Investor has to bear in mind that there is always a trade – off between risk and return. Risk averse investors would prefer allocations as suggested by Scenario 3 and Scenario 4, whereas Risk takers would prefer Scenario 1 and Scenario 2. However the choices are subjective. The choice of proportion of allocation depends on the risk appetite and investment objectives.In order to achieve a fair balance between return to risk trade – off, it is advisable to opt for optimal proportionate allocation with the objective of maximizing return to risk ratio or Sharpe Ratio (Scenario 5).

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